3.5 ERRORS OF UNCERTAINTY

General
The principle of determining the uncertainty of any type of scientific or engineering based measurement follows similar guidelines. There are a number of guidelines available but because these have to cater for any type of measurement, they tend to be all-encompassing and it is not always quite so obvious how to adapt it to a particular requirement.

The response of many people in the Weighing Industry when faced with tackling measurement uncertainty is "why bother, I already use calibrated weights"?

As a purely hypothetical illustration consider the following; You were caught speeding in your car by a policeman with a hand-held speed gun and then prosecuted for travelling at 49mph in a 40mph limited area. Following this prosecution, you enquire of the accuracy of the hand held speed gun and are told ±10%. You assume fair play as you were accused of travelling well over 44mph.

However on making further enquires, you discover the following:

- The gun was calibrated in a test laboratory strapped to a bench and hand operation can add a further ±4mph uncertainty to the readings;
- The specification of the hand gun quotes a possible error of ±1% per °C. As the calibration was performed at 20 °C and the temperature at the time of the offence was measured at 10°C, the measurement has a further uncertainty of ±10%.

Are you still satisfied the prosecution was fair?

What Constitutes an Uncertainty?
The obvious answer has to be anything that can influence the measurement process. However, what you actually need to include in the uncertainty calculation will depend upon the significance of the effect upon the measurement.

There are some uncertainties less obvious than others. For instance, if you use calibrated weights the drift between calibrations can be significant. In order to account for this drift, the actual value of the weight needs to be recorded at the time of calibration and a record kept. From this record the drift can be calculated.

A fairly comprehensive list of the types of uncertainty associated with the weighing process can be found in Measurement Good Practice Guide No. 71, 2004, 'The Measurement of Mass and Weight' published by The Institute of Measurement and Control and available as a download from the NPL website:

http://publications.npl.co.uk/npl_web/pdf/mgpg71.pdf.

Make a list of all the possible effects that can influence the measurement, including the value of the effect. This is often expressed in parts per million (ppm).

Rank the uncertainty types in their order of influence.

Some effects will be more dominant than others. So there will be major influences and minor ones.

There is a school of opinion that because of the summation process used in the uncertainty calculation, anything less than one tenth of the most major influence will have an insignificant effect on the final result. Some people will of course disagree and include everything.

As a practical example: If you are vessel weighing, the influence of both connecting pipes and wind speed could be greater than 1,000ppm. So you probably need not consider the effect of gravity variation due to the Lunar phase changes which will be in the order of 0.1ppm if the weighing system is measuring force (e.g. load cells). However, if you have a force generating machine in a National Standards Laboratory and were trying to achieve an uncertainty of better than 1ppm, the Lunar gravity change would become significant.

Different types of uncertainty

The collection and summation of these uncertainties use statistically-based processes. There are some quite comprehensive guidelines published that describe the decision making and statistical processes in detail. Two are listed below:

"EA-4/02, 1999, Expressions of the Uncertainty of Measurements in Calibration" published by the European co-operation for Accreditation and available on their website:

"M3003 The Expression of Uncertainty and Confidence in Measurement" published by UKAS and available on their website:

There are two additional guides by UKAS that discuss the subject: LAB 12 The Expression of Uncertainty in Testing and LAB 14 Calibration of Weighing Machines, both also available from the UKAS website above.

Although the above publications give a comprehensive guide, a brief overview of the process is given below.

The uncertainties can be banded into two types:-

The ones that follow a normal type distribution and the ones that don't. These are known as type A and type B. A simplified descriptive example, showing the different types of measurement uncertainty, is shown below.

Standard Uncertainty

1) Standard Uncertainty of the **resolution** of the indicator $\mathbf{u_{res}}$

This is a rectangular distribution and therefore a type B.

$$u_{res} = \frac{a}{2\sqrt{3}}$$

Where a is the resolution of the indicator

2) Standard Uncertainty of the **repeatability** of the indicator $\mathbf{u_{rep}}$

This is a normal distribution and therefore a type A

$$u_{rep} = \frac{1}{\sqrt{n}} \left(\frac{100}{d_m} \sqrt{\frac{1}{(n-1)} \sum_{j=1}^{j=n} (d_J - d_m)^2} \right)$$

Where

n = N° of readings

d_J = value for each test

d_m = mean of value

This part of the above equation:-

$$\sum_{j=1}^{j=n} (d_J - d_m)^2$$

may look complicated but it is only the mean of all the load readings at this particular point (say 2kg on the way up), subtracted from each individual reading at this load value. This is then squared. This process is repeated for all the individual readings at this point. All these squared readings are then added together.

If you don't want the answer in percent leave out the100. If you want the answer in ppm, replace the 100 by 10^6.

3) Standard Uncertainty of the **applied force u_{std}**

This is often a normal distribution and therefore a type A. Sum the uncertainties quoted on the calibration certificate(s) for all the weights used.

4) Uncorrected **drift of the standard** weight(s) since their last calibration **u_{drift}**

This is a rectangular distribution and therefore a type B. Usually at least 1x the uncertainty of calibration for each weight used; but the actual drift can be estimated from the trends of the calibration histories.

5) Air buoyancy correction **u_{AB}**

This is a rectangular distribution and therefore a type B. If the span of the weighing instrument is adjusted before calibration then uncertainty limits of 1ppm for stainlessteel weights or 3ppm for cast iron weights could be used. Note: the ppm values are derived from the instrument's range under test (often the capacity of the instrument).

Combining Uncertainties

The **combined standard uncertainty** (u_c) is calculated from the square root of the sum of the squares of the individual standard uncertainties.

$$u_c = \sqrt{u_{res}^2 + u_{rep}^2 + u_{std}^2 + u_{drift}^2 + u_{AB}^2}$$

Expanded Uncertainty (u_{exp})

The calculation of the expanded uncertainty is simply the coverage factor (k) multiplied by the combined uncertainty (u_c)

$u_{exp} = k \times u_c$

The generally accepted practice is to use a coverage factor of k = 2 and use this to calculate the expanded uncertainty. Normally this will give a confidence level of approximately 95%.

Note: The value for the coverage factor (k) may have to be modified, if the random contributions (usually repeatability) are relatively large compared with the other contributions. Modification of the coverage factor (k) is dealt with in UKAS document M3003.

The above notes are for guidance only. Each measurement process is likely to be unique and will therefore require its own tailor made uncertainty contributions.